

On electromagnetic interactions for massive mixed symmetry field

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Abstract

In this paper we investigate electromagnetic interactions for simplest massive mixed symmetry field. Using frame-like gauge invariant formulation we extend Fradkin-Vasiliev procedure, initially proposed for investigation of gravitational interactions for massless particles in AdS space, to the case of electromagnetic interactions for massive particles leaving in $(A)dS$ space with arbitrary value of cosmological constant including flat Minkowski space. At first, as an illustration of general procedure, we re-derive our previous results on massive spin 2 electromagnetic interactions and then we apply this procedure to massive mixed symmetry field. These two cases are just the simplest representatives of two general class of fields, namely completely symmetric and mixed symmetry ones, and it is clear that the results obtained admit straightforward generalization to higher spins as well.

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Introduction

It has been known since a long time that it is not possible to construct standard gravitational interaction for massless higher spin $s \geq 5/2$ particles in flat Minkowski space (see [1] and references therein). At the same time, it has been shown [2, 3] that this task indeed has a solution in $(A)dS$ space with non-zero cosmological term. The reason is that gauge invariance, that turns out to be broken when one replaces ordinary partial derivatives by the gravitational covariant ones, could be restored with the introduction of higher derivative corrections containing gauge invariant Riemann tensor. These corrections have coefficients proportional to inverse powers of cosmological constant so that such theories do not have naive flat limit. However it is perfectly possible, for cubic vertices, to have a limit where both cosmological term and gravitational coupling constant simultaneously go to zero in such a way that only interactions with highest number of derivatives survive [4, 5].

Besides gravitational interaction one more classical and important test for any higher spin theory is electromagnetic interaction. The problem of switching on such interaction for massless higher spin particles looks very similar to the problem with gravitational interactions. Namely, if one replaces ordinary partial derivatives by the gauge covariant ones the resulting Lagrangian loses its gauge invariance and this non-invariance (arising due to non-commutativity of covariant derivatives) is proportional to field strength of vector field. In this, for the massless fields with $s \geq 3/2$ in flat Minkowski space there is no possibility to restore gauge invariance by adding non-minimal terms to Lagrangian and/or modifying gauge transformations. But such restoration becomes possible if one goes to $(A)dS$ space with non-zero cosmological constant. By the same reason, as in the gravitational case, such theories do not have naive flat limit, but it is possible to consider a limit where both cosmological constant and electric charge simultaneously go to zero so that only highest derivative non-minimal terms survive [5, 6].

It is natural to suggest that in any realistic higher spin theory (like in superstring) most of higher spin particles must be massive and their gauge symmetries spontaneously broken. As is well known, for massive higher spin particles any attempt to switch on standard minimal gravitational or electromagnetic interactions spoils a consistency of the theory leading first of all to appearance of non-physical degrees of freedom and/or non-causality. But having in our disposal mass m as a dimensionfull parameter even in a flat Minkowski space we can try to restore consistency of the theory by adding to Lagrangian non-minimal terms containing the linearized Riemann tensor (e/m field strength). Naturally such terms will have coefficients proportional to inverse powers of mass m so that the theory will not have naive massless limit. However, it is natural to suggest that there exists a limit where both mass and gravitational coupling constant (electric charge) simultaneously go to zero so that only some interactions containing Riemann tensor (e/m field strength) survive. Note that such a picture agrees with the results obtained from (super)strings [7, 8, 9].

As is well known, in four dimensions to describe all possible higher spins it is enough to consider completely symmetric (spin-)tensors only. But in dimensions greater than four it is necessary to consider mixed symmetry (spin-)tensors as well. There exists a number of works devoted to investigation of possible interactions for such fields [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21], but till now most of them produce so called "no-go" results (note however light-cone results [22] and recent work [23]).

In this paper we investigate electromagnetic interactions for simplest massive mixed symmetry field ("hook"). In this, we extend Fradkin-Vasiliev procedure, initially proposed for investigation of gravitational interactions for massless particles in AdS space and then successfully applied to other massless higher spin actions [24, 25, 26, 23], to the case of electromagnetic interactions for massive particles leaving in $(A)dS$ space with arbitrary value of cosmological constant including flat Minkowski space. One of the essential ingredients of the procedure is the frame-like formalism, initially introduced for completely symmetric (spin-)tensors [27, 28, 29] (see also [30, 31]) and then extended to the case of massless [32, 33, 34, 35, 36, 37, 38, 39] and massive [40, 41, 42, 43] mixed symmetry (spin-)tensors. Briefly, the procedure can be described as follows.

- Take as input frame-like gauge invariant formulation for massive particle.
- Using explicit form of gauge transformations construct for all fields (both physical and auxiliary) gauge invariant objects which in what follows we will call "curvatures" though there will be one, two and three forms among them.
- Rewrite free Lagrangian as an expression quadratic in these curvatures. Note that in general even if one requires that all higher derivatives terms were absent and kinetic terms were diagonal one still faces some ambiguity in the choice of coefficients.
- Find deformations of these curvatures supplemented with appropriate corrections to gauge transformations such that their gauge variations be proportional to free curvatures. In the case of electromagnetic interactions the only non-trivial task is to find deformation for electromagnetic field strength while all we have to do with other curvatures is to replace AdS covariant derivatives with the fully covariant ones.
- In the free Lagrangian replace free curvatures with the deformed ones and adjust free parameters so that all variations vanish on-shell. Note that at least in some cases this resolves ambiguity with the parameters in the free Lagrangian as well.
- Determine corrections to gauge transformations such that all gauge variations vanish off-shell.

The plan of the paper is simple. In Section 1, as an illustration of general procedure, we re-derive our previous results on massive spin 2 electromagnetic interactions [44]. All physical results are the same, but derivation becomes much more simple and transparent. Then in Section 2 we apply this procedure to the electromagnetic interactions for simplest massive mixed symmetry tensor.

Notations and conventions. We work in $(A)dS$ space with $d \geq 4$ dimensions. We will use notation $e_\mu{}^a$ for background (non-dynamical) frame of $(A)dS$ space and D_μ for $(A)dS$ covariant derivatives normalized so that

$$[D_\mu, D_\nu]\xi^a = -\kappa e_{[\mu}{}^a \xi_{\nu]}, \quad \kappa = \frac{2\lambda}{(d-1)(d-2)}$$

We use Greek letters for world indices and Latin letters for local ones. Surely, using frame $e_\mu{}^a$ and its inverse $e^\mu{}_a$ one can freely convert world indices into local ones and vice-verse

and we indeed will use such conversion whenever convenient. But separation of world and local indices plays very important role in a frame-like formalism. In particular, all terms in the Lagrangians can be written as a product of forms, i.e. as expressions completely antisymmetric on world indices and this property greatly simplifies all calculations. For that purpose we will often use notations $\{\overset{\mu\nu}{ab}\} = e^\mu{}_a e^\nu{}_b - e^\nu{}_a e^\mu{}_b$ and so on.

1 Massive spin 2

In this section, as an illustration of general procedure, we re-derive our previous results on spin 2 electromagnetic interactions [6, 44].

1.1 Lagrangian and gauge transformations

Frame-like gauge invariant formulations for massive spin 2 particles [40, 30] requires three pairs of physical and auxiliary fields: $(\omega_\mu{}^{ab}, f_\mu{}^a)$, (B^{ab}, B_μ) and (π^a, φ) . The free Lagrangian in the constant curvature space with arbitrary value of cosmological constant has the form:

$$\begin{aligned} \mathcal{L}_0 = & \frac{1}{2} \{\overset{\mu\nu}{ab}\} \omega_\mu{}^{ac} \omega_\nu{}^{bc} - \frac{1}{2} \{\overset{\mu\nu\alpha}{abc}\} \omega_\mu{}^{ab} D_\nu f_\alpha{}^c + \frac{1}{4} B_{ab}{}^2 - \frac{1}{2} \{\overset{\mu\nu}{ab}\} B^{ab} D_\mu B_\nu - \\ & - \frac{1}{2} \pi_a{}^2 + e^\mu{}_a \pi^a D_\mu \varphi + m[\{\overset{\mu\nu}{ab}\} \omega_\mu{}^{ab} B_\nu + e^\mu{}_a B^{ab} f_\mu{}^b] - \tilde{M} e^\mu{}_a \pi^a B_\mu + \\ & + M^2 \{\overset{\mu\nu}{ab}\} f_\mu{}^a f_\nu{}^b - m \tilde{M} e^\mu{}_a f_\mu{}^a \varphi + \frac{d}{(d-2)} m^2 \varphi^2 \end{aligned} \quad (1)$$

Here $M^2 = m^2 - \frac{\kappa(d-2)}{2}$, $\tilde{M} = 2\sqrt{\frac{(d-1)}{(d-2)}} M$.

This Lagrangian is invariant under the following gauge transformations:

$$\begin{aligned} \delta_0 f_\mu{}^a &= D_\mu \xi^a + \eta_\mu{}^a + \frac{2m}{(d-2)} e_\mu{}^a \Lambda, & \delta_0 \omega_\mu{}^{ab} &= D_\mu \eta^{ab} - \frac{2M^2}{(d-2)} e_\mu{}^{[a} \xi^{b]} \\ \delta_0 B_\mu &= D_\mu \Lambda + m \xi_\mu, & \delta_0 B^{ab} &= -2m \eta^{ab}, & \delta_0 \varphi &= \tilde{M} \Lambda, & \delta_0 \pi^a &= -m \tilde{M} \xi^a \end{aligned} \quad (2)$$

Note that in a de Sitter space ($\kappa > 0$) there exists so called partially massless limit [45, 46, 47, 48, 49, 50] where spin 0 component completely decouples leaving us with the Lagrangian

$$\begin{aligned} \mathcal{L}_0 = & \frac{1}{2} \{\overset{\mu\nu}{ab}\} \omega_\mu{}^{ac} \omega_\nu{}^{bc} - \frac{1}{2} \{\overset{\mu\nu\alpha}{abc}\} \omega_\mu{}^{ab} D_\nu f_\alpha{}^c + \frac{1}{4} B_{ab}{}^2 - \frac{1}{2} \{\overset{\mu\nu}{ab}\} B^{ab} D_\mu B_\nu + \\ & + m[\{\overset{\mu\nu}{ab}\} \omega_\mu{}^{ab} B_\nu + e^\mu{}_a B^{ab} f_\mu{}^b] \end{aligned} \quad (3)$$

which is invariant under the following gauge transformations

$$\begin{aligned} \delta_0 f_\mu{}^a &= D_\mu \xi^a + \eta_\mu{}^a + \frac{2m}{(d-2)} e_\mu{}^a \Lambda, & \delta_0 \omega_\mu{}^{ab} &= D_\mu \eta^{ab} \\ \delta_0 B_\mu &= D_\mu \Lambda + m \xi_\mu, & \delta_0 B^{ab} &= -2m \eta^{ab} \end{aligned} \quad (4)$$

1.2 Gauge invariant objects

For all six fields (both physical and auxiliary) we can construct corresponding gauge invariant object ("curvature"):

$$\begin{aligned}
\mathcal{F}_{\mu\nu}{}^{ab} &= D_{[\mu}\omega_{\nu]}{}^{ab} - \frac{m}{(d-2)}e_{[\mu}{}^{[a}B_{\nu]}{}^{b]} - \frac{2M^2}{(d-2)}e_{[\mu}{}^{[a}f_{\nu]}{}^{b]} + \frac{2m\tilde{M}}{(d-1)(d-2)}e_{[\mu}{}^a e_{\nu]}{}^b \varphi \\
T_{\mu\nu}{}^a &= D_{[\mu}f_{\nu]}{}^a - \omega_{[\mu,\nu]}{}^a + \frac{2m}{(d-2)}e_{[\mu}{}^a B_{\nu]} \\
\mathcal{B}_\mu{}^{ab} &= D_\mu B^{ab} + 2m\omega_\mu{}^{ab} - \frac{\tilde{M}}{(d-1)}e_\mu{}^{[a}\pi^{b]} \\
\mathcal{B}_{\mu\nu} &= D_{[\mu}B_{\nu]} - B_{\mu\nu} - mf_{[\mu,\nu]} \\
\Pi_\mu{}^a &= D_\mu\pi^a + \frac{\tilde{M}}{2}B_\mu{}^a + m\tilde{M}f_\mu{}^a - \frac{2m^2}{(d-2)}e_\mu{}^a \varphi \\
\Phi_\mu &= D_\mu\varphi - \pi_\mu - \tilde{M}B_\mu
\end{aligned} \tag{5}$$

As it has been shown in [31] for arbitrary spin massive particles the free Lagrangian can be rewritten as expression quadratic in these gauge invariant curvatures. For the case at hands the most general such Lagrangian can be written as follows:

$$\begin{aligned}
\mathcal{L}_0 &= a_1 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \mathcal{F}_{\mu\nu}{}^{ab} \mathcal{F}_{\alpha\beta}{}^{cd} + a_2 \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \mathcal{F}_{\mu\nu}{}^{ab} \Pi_\alpha{}^c + a_3 \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \mathcal{B}_\mu{}^{ac} \mathcal{B}_\nu{}^{bc} + a_4 \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \Pi_\mu{}^a \Pi_\nu{}^b + \\
&\quad + b_1 \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} T_{\mu\nu}{}^a \mathcal{B}_\alpha{}^{bc} + b_2 \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \mathcal{B}_\mu{}^{ab} \Phi_\nu
\end{aligned} \tag{6}$$

Note that even if we require that higher derivatives terms were absent and kinetic terms were diagonal there still exists some ambiguity in the choice of coefficients. In what follows (see below) we will use the following simple concrete choice:

$$\mathcal{L}_0 = a_1 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \mathcal{F}_{\mu\nu}{}^{ab} \mathcal{F}_{\alpha\beta}{}^{cd} + a_3 \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \mathcal{B}_\mu{}^{ac} \mathcal{B}_\nu{}^{bc} + b_2 \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \mathcal{B}_\mu{}^{ab} \Phi_\nu \tag{7}$$

where

$$\tilde{a}_1 = \frac{16(d-3)}{(d-2)}a_1 = \frac{1}{4M^2}, \quad a_3 = \frac{1}{8M^2}, \quad b_2 = -\frac{1}{2\tilde{M}}$$

Note that up to normalization it is just the Ponomarev-Vasiliev Lagrangian (4.20) in [31].

1.3 Deformations

Now let us turn to the electromagnetic interactions. We prefer to work with real fields, so from now on all components of massive spin 2 will become doublets $f_\mu{}^a \rightarrow f_\mu{}^{a,i}$, $i = 1, 2$ and so on. Our first task — to find appropriate deformations of all gauge invariant curvatures given above as well of electromagnetic field strength. As for massive spin 2 curvatures, the answer appears to be very simple: all we have to do is to replace $(A)dS$ covariant derivatives by fully covariant ones:

$$D_\mu \implies \nabla_\mu{}^{ij} = \delta^{ij}D_\mu + e_0\varepsilon^{ij}A_\mu$$

where e_0 — electric charge. As for the electromagnetic field strength, having in our disposal explicit expressions for curvatures and gauge transformations it is not hard to find desired result.

Let us consider the following ansatz for such deformation:

$$\hat{F}_{\mu\nu} = F_{\mu\nu} + a_0 \varepsilon^{ij} [\omega_{[\mu}{}^{ab,i} \omega_{\nu]}{}^{ab,j} + \alpha_1 B_{[\mu}{}^{a,i} B_{\nu]}{}^{a,j} + \alpha_2 f_{[\mu}{}^{a,i} f_{\nu]}{}^{a,j} + \alpha_3 B_{[\mu}{}^i B_{\nu]}^j + \alpha_4 B_{\mu\nu}{}^i \varphi^j]$$

Its variation under η^{ab} gauge transformations has the form:

$$\delta \hat{F}_{\mu\nu} = 2a_0 \varepsilon^{ij} [-D_{[\mu} \eta^{ab,j} \omega_{\nu]}{}^{ab,i} - 2m\alpha_1 B_{[\mu}{}^{a,i} \eta_{\nu]}{}^{a,j} + \alpha_2 f_{[\mu}{}^{a,i} \eta_{\nu]}{}^{a,j} + m\alpha_4 \varphi^i \eta_{\mu\nu}{}^j]$$

Now we introduce correction to e/m field gauge transformations:

$$\delta A_\mu = 2a_0 \varepsilon^{ij} \omega_\mu{}^{ab,i} \eta^{ab,j}$$

As a result we obtain:

$$\delta \hat{F}_{\mu\nu} = 2a_0 \varepsilon^{ij} [D_{[\mu} \omega_{\nu]}{}^{ab,i} \eta^{ab,j} - 2m\alpha_1 B_{[\mu}{}^{a,i} \eta_{\nu]}{}^{a,j} + \alpha_2 f_{[\mu}{}^{a,i} \eta_{\nu]}{}^{a,j} + m\alpha_4 \varphi^i \eta_{\mu\nu}{}^j]$$

Comparing this expression with

$$\begin{aligned} 2a_0 \varepsilon^{ij} \mathcal{F}_{\mu\nu}{}^{ab,i} \eta^{ab,j} &= 2a_0 \varepsilon^{ij} [D_{[\mu} \omega_{\nu]}{}^{ab,i} - \frac{2m}{(d-2)} e_{[\mu}{}^a B_{\nu]}{}^{b,i} - \frac{4M^2}{(d-2)} e_{[\mu}{}^a f_{\nu]}{}^{b,i} + \\ &\quad + \frac{2m\tilde{M}}{(d-1)(d-2)} e_{[\mu}{}^a e_{\nu]}{}^b \varphi^i] \eta^{ab,j} \end{aligned}$$

we see that we have to put

$$\alpha_1 = -\frac{1}{(d-2)}, \quad \alpha_2 = \frac{4M^2}{(d-2)}, \quad \alpha_4 = \frac{4\tilde{M}}{(d-1)(d-2)}$$

Similarly, variation under ξ^a transformations:

$$\delta \hat{F}_{\mu\nu} = 2a_0 \varepsilon^{ij} [-\alpha_2 D_{[\mu} \xi^{a,j} f_{\nu]}{}^{a,i} - \frac{4M^2}{(d-2)} \omega_{[\mu,\nu]}{}^{a,i} \xi^{a,j} + m\alpha_3 B_{[\mu}{}^i \xi_{\nu]}^j]$$

together with corresponding correction

$$\delta A_\mu = 2a_0 \alpha_2 \varepsilon^{ij} f_\mu{}^{a,i} \xi^{a,j}$$

produce

$$\delta \hat{F}_{\mu\nu} = 2a_0 \varepsilon^{ij} [\alpha_2 \xi^{a,j} D_{[\mu} f_{\nu]}{}^{a,i} - \frac{4M^2}{(d-2)} \omega_{[\mu,\nu]}{}^{a,i} \xi^{a,j} + m\alpha_3 B_{[\mu}{}^i \xi_{\nu]}^j]$$

Comparing this expression with

$$2a_0 \alpha_2 \varepsilon^{ij} T_{\mu\nu}{}^{a,i} \xi^{a,j} = 2a_0 \alpha_2 \varepsilon^{ij} [D_{[\mu} f_{\nu]}{}^{a,i} - \omega_{[\mu,\nu]}{}^{a,i} + \frac{2m}{(d-2)} e_{[\mu}{}^a B_{\nu]}^i] \xi^{a,j}$$

we get

$$\alpha_3 = -\frac{8M^2}{(d-2)^2}$$

All the unknown coefficients are already fixed, but as a useful check one can easily see that variation under Λ transformations

$$\delta \hat{F}_{\mu\nu} = a_0 \varepsilon^{ij} [-2\alpha_3 D_{[\mu} \Lambda^j B_{\nu]}^i + \frac{4m\alpha_2}{(d-2)} f_{[\mu,\nu]}^i \Lambda^j + \tilde{M} \alpha_4 B_{\mu\nu}^i \Lambda^j]$$

together with

$$\delta A_\mu = 2\alpha_3 a_0 \varepsilon^{ij} B_\mu^i \Lambda^j$$

give the desired result:

$$\delta \hat{F}_{\mu\nu} = a_0 \varepsilon^{ij} [2\alpha_3 D_{[\mu} B_{\nu]}^i \Lambda^j + \frac{4m\alpha_2}{(d-2)} f_{[\mu,\nu]}^i \Lambda^j + \tilde{M} \alpha_4 B_{\mu\nu}^i \Lambda^j] = -\frac{16M^2 a_0}{(d-2)^2} \varepsilon^{ij} \mathcal{B}_{\mu\nu}^i \Lambda^j$$

Collecting all pieces together, we obtain finally

$$\begin{aligned} \hat{F}_{\mu\nu} = & F_{\mu\nu} + a_0 \varepsilon^{ij} [\omega_{[\mu}^{ab,i} \omega_{\nu]}^{ab,j} - \frac{1}{(d-2)} B_{[\mu}^{a,i} B_{\nu]}^{a,j} + \frac{4M^2}{(d-2)} f_{[\mu}^{a,i} f_{\nu]}^{a,j} - \\ & - \frac{8M^2}{(d-2)^2} B_{[\mu}^i B_{\nu]}^j + \frac{4\tilde{M}}{(d-1)(d-2)} B_{\mu\nu}^i \varphi^j] \end{aligned} \quad (8)$$

In this, under massive spin 2 gauge transformations supplemented with the following corrections to e/m field gauge transformations:

$$\delta A_\mu = 2a_0 \varepsilon^{ij} [\omega_\mu^{ab,i} \eta^{ab,j} + \frac{4M^2}{(d-2)} f_\mu^{a,i} \xi^{a,j} - \frac{8M^2}{(d-2)^2} B_\mu^i \Lambda^j] \quad (9)$$

this tensor transforms as:

$$\delta \hat{F}_{\mu\nu} = 2a_0 \varepsilon^{ij} [\mathcal{F}_{\mu\nu}^{ab,i} \eta^{ab,j} + \frac{4M^2}{(d-2)} T_{\mu\nu}^{a,i} \xi^{a,j} - \frac{8M^2}{(d-2)^2} \mathcal{B}_{\mu\nu}^i \Lambda^j] \quad (10)$$

Now, following general procedure, we consider Lagrangian:

$$\mathcal{L}_0 = a_1 \left\{ \frac{\mu\nu\alpha\beta}{abcd} \right\} \hat{\mathcal{F}}_{\mu\nu}^{ab} \hat{\mathcal{F}}_{\alpha\beta}^{cd} + a_3 \left\{ \frac{\mu\nu}{ab} \right\} \hat{\mathcal{B}}_\mu^{ac} \hat{\mathcal{B}}_\nu^{bc} + b_2 \left\{ \frac{\mu\nu}{ab} \right\} \hat{\mathcal{B}}_\mu^{ab} \hat{\Phi}_\nu - \frac{1}{4} \hat{F}_{\mu\nu}^2 \quad (11)$$

1.4 Gauge invariance

Now we have to calculate variations of this Lagrangian in the first non-trivial approximation and to adjust parameters so that all such variations vanish on-shell. This in turn would imply that they can be compensated by appropriate corrections to gauge transformations.

η^{ab} transformations. In this case we obtain the following variations of our Lagrangian:

$$\begin{aligned} \delta_\eta \mathcal{L} = & -4a_1 e_0 \varepsilon^{ij} \left\{ \frac{\mu\nu}{ab} \right\} [4\mathcal{F}_{\mu\nu}^{ac,i} \eta^{bd,j} - \mathcal{F}_{\mu\nu}^{ab,i} \eta^{cd,j}] F^{cd} + \\ & + \varepsilon^{ij} \mathcal{F}_{\mu\nu}^{ab,i} [8a_1 e_0 \eta^{\mu\nu,j} F^{ab} - a_0 F^{\mu\nu} \eta^{ab,j}] \end{aligned} \quad (12)$$

Terms in the first line clearly vanish on-shell being proportional to free f_μ^a equations. Now, using explicit expressions for free curvatures it is not hard to check that the following identity holds:

$$\mathcal{F}_{[\mu\nu,\alpha]}^a = -D_{[\mu} T_{\nu\alpha]}^a - \frac{2m}{(d-2)} e_{[\mu}^a \mathcal{B}_{\nu\alpha]} \quad (13)$$

Thus $\mathcal{F}_{[\mu\nu,\alpha]}^a$ vanish on-shell and as a consequence we obtain

$$\mathcal{F}_{[\mu\nu,\alpha]}^a = 0 \quad \implies \quad \mathcal{F}_{ab,cd} = \mathcal{F}_{cd,ab}$$

So if we set $a_0 = 8a_1e_0$ then all η^{ab} variations vanish on shell and can be compensated by appropriate corrections to gauge transformations. For simplicity we restrict ourselves with the corrections to gauge transformations for physical fields only which in this case have the form:

$$\delta_1 f_\mu^{a,i} = 4a_0 \varepsilon^{ij} [F_\mu^b \eta^{ab,j} - \frac{1}{2(d-2)} e_\mu^a (F\eta)^j] \quad (14)$$

ξ^a **transformations.** Here we obtain simply

$$\delta_\xi \mathcal{L} = -\frac{4M^2}{(d-2)} a_0 \varepsilon^{ij} F^{\mu\nu} T_{\mu\nu}^{a,i} \xi^{a,j}$$

This term vanish on-shell, in this it can be compensated by corresponding corrections to ω_μ^{ab} transformations.

Λ **transformations.** Here we get

$$\delta_\Lambda \mathcal{L} = -\frac{8M^2}{(d-2)^2} a_0 \varepsilon^{ij} F^{\mu\nu} \mathcal{B}_{\mu\nu}^i \Lambda^j$$

This term also vanish on-shell and can be compensated by corresponding corrections to B^{ab} transformations.

1.5 Results

Thus we have seen that the following Lagrangian

$$\mathcal{L}_0 = a_1 \left\{ \frac{\mu\nu\alpha\beta}{abcd} \right\} \hat{\mathcal{F}}_{\mu\nu}^{ab} \hat{\mathcal{F}}_{\alpha\beta}^{cd} + a_3 \left\{ \frac{\mu\nu}{ab} \right\} \hat{\mathcal{B}}_\mu^{ac} \hat{\mathcal{B}}_\nu^{bc} + b_2 \left\{ \frac{\mu\nu}{ab} \right\} \hat{\mathcal{B}}_\mu^{ab} \hat{\Phi}_\nu - \frac{1}{4} \hat{F}_{\mu\nu}^2 \quad (15)$$

is indeed gauge invariant in the linear approximation provided we introduce corresponding corrections to gauge transformations

$$\begin{aligned} \delta A_\mu &= 2a_0 \varepsilon^{ij} [\omega_\mu^{ab,i} \eta^{ab,j} + \frac{4M^2}{(d-2)} f_\mu^{a,i} \xi^{a,j} - \frac{8M^2}{(d-2)^2} B_\mu^i \Lambda^j] \\ \delta f_\mu^{a,i} &= 4a_0 \varepsilon^{ij} [F_\mu^b \eta^{ab,j} - \frac{1}{2(d-2)} e_\mu^a (F\eta)^j] \end{aligned} \quad (16)$$

Note that in agreement with our previous results in [44] it is impossible to take partially massless limit $M \rightarrow 0$ in de Sitter space without switching off minimal e/m interactions. At the same time, nothing prevents us from considering massless limit in anti de Sitter space $m = 0$, $M^2 = -\frac{\kappa(d-2)}{2}$. In this limit we obtain simple Lagrangian

$$\mathcal{L} = a_1 \left\{ \frac{\mu\nu\alpha\beta}{abcd} \right\} \mathcal{F}_{\mu\nu}^{ab} \mathcal{F}_{\alpha\beta}^{cd} - \frac{1}{4} \hat{F}_{\mu\nu}^2 \quad (17)$$

where

$$\begin{aligned} \mathcal{F}_{\mu\nu}^{ab} &= \nabla_{[\mu} \omega_{\nu]}^{ab} - \frac{2M^2}{(d-2)} e_{[\mu}^{[a} f_{\nu]}^{b]} \\ \hat{F}_{\mu\nu} &= F_{\mu\nu} + a_0 \varepsilon^{ij} [\omega_{[\mu}^{ab,i} \omega_{\nu]}^{ab,j} + \frac{4M^2}{(d-2)} f_{[\mu}^{a,i} f_{\nu]}^{a,j}] \end{aligned} \quad (18)$$

2 Mixed symmetry tensor

In this section we apply the same procedure to construct electromagnetic interactions for simplest massive mixed symmetry tensor ("hook").

2.1 Lagrangian and gauge transformations

In this case frame-like gauge invariant description [40, 41] requires four pairs of physical and auxiliary fields: $(\Omega_\mu^{abc}, \Phi_{\mu\nu}^a)$, $(\omega_\mu^{ab}, h_\mu^a)$, $(C^{abc}, C_{\mu\nu})$ and (B^{ab}, B_μ) . Free Lagrangian has the form:

$$\begin{aligned}
\mathcal{L}_0 = & -\frac{3}{4} \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \Omega_\mu^{acd} \Omega_\nu^{bcd} + \frac{1}{4} \{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \} \Omega_\mu^{abc} D_\nu \Phi_{\alpha\beta}^d + \\
& + \frac{1}{2} \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \omega_\mu^{ac} \omega_\nu^{bc} - \frac{1}{2} \{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \} \omega_\mu^{ab} D_\nu f_\alpha^c - \\
& - \frac{1}{6} C_{abc}^2 + \frac{1}{6} \{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \} C^{abc} D_\mu C_{\nu\alpha} + \frac{1}{4} B_{ab}^2 - \frac{1}{2} \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} B^{ab} D_\mu B_\nu + \\
& + m_1 [\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \Omega_\mu^{abc} f_\nu^c + \{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \} \omega_\mu^{ab} \Phi_{\nu\alpha}^c] + \\
& + m_2 [\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \} \Omega_\mu^{abc} C_{\nu\alpha} + \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} C^{abc} \Phi_{\mu\nu}^c] + \\
& + 2\tilde{m}_2 [\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \omega_\mu^{ab} B_\nu + \{ \begin{smallmatrix} \mu \\ a \end{smallmatrix} \} B^{ab} f_\mu^b] + \tilde{m}_1 \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} B^{ab} C_{\mu\nu}
\end{aligned} \tag{19}$$

parameters $m_{1,2}$ satisfy a relation

$$8m_1^2 - 24m_2^2 = -3(d-3)\kappa$$

while $\tilde{m}_{1,2} = \sqrt{\frac{(d-2)}{(d-3)}} m_{1,2}$. This Lagrangian is invariant under the following set of gauge transformations:

$$\begin{aligned}
\delta_0 \Phi_{\mu\nu}^a &= D_{[\mu} z_{\nu]}^a + \eta_{\mu\nu}^a + \frac{2m_1}{3(d-3)} e_{[\mu}^a \xi_{\nu]} + \frac{4m_2}{(d-3)} e_{[\mu}^a \zeta_{\nu]} \\
\delta_0 \Omega_\mu^{abc} &= D_\mu \eta^{abc} + \frac{4m_1}{3(d-3)} e_\mu^{[a} \eta^{bc]} \\
\delta_0 f_\mu^a &= D_\mu \xi^a + \eta_\mu^a + 4m_1 z_\mu^a + \frac{4\tilde{m}_2}{(d-2)} e_\mu^a \Lambda \\
\delta_0 \omega_\mu^{ab} &= D_\mu \eta^{ab} - 2m_1 \eta_\mu^{ab} \\
\delta_0 C_{\mu\nu} &= D_{[\mu} \zeta_{\nu]} - 2m_2 z_{[\mu\nu]}, \quad \delta_1 C^{abc} = 6m_2 \eta^{abc} \\
\delta_0 B_\mu &= D_\mu \Lambda + 2\tilde{m}_2 \xi_\mu + 4\tilde{m}_1 \zeta_\mu, \quad \delta_0 B^{ab} = -4\tilde{m}_2 \eta^{ab}
\end{aligned} \tag{20}$$

As the relation on the parameters $m_{1,2}$ clearly shows for non-zero values of cosmological constant κ it is not possible to set both m_1 and m_2 equal to zero simultaneously. In AdS space ($\kappa < 0$) one can set $m_2 = 0$. In this, the whole system decomposes into two disconnected subsystems. One of them with the Lagrangian and gauge transformations:

$$\begin{aligned}
\mathcal{L}_0 = & -\frac{3}{4} \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \Omega_\mu^{acd} \Omega_\nu^{bcd} + \frac{1}{4} \{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \} \Omega_\mu^{abc} D_\nu \Phi_{\alpha\beta}^d + \\
& + \frac{1}{2} \{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \omega_\mu^{ac} \omega_\nu^{bc} - \frac{1}{2} \{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \} \omega_\mu^{ab} D_\nu f_\alpha^c - \\
& + m_1 [\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \} \Omega_\mu^{abc} f_\nu^c + \{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \} \omega_\mu^{ab} \Phi_{\nu\alpha}^c]
\end{aligned} \tag{21}$$

$$\begin{aligned}
\delta_0 \Phi_{\mu\nu}{}^a &= D_{[\mu} z_{\nu]}{}^a + \eta_{\mu\nu}{}^a + \frac{2m_1}{3(d-3)} e_{[\mu}{}^a \xi_{\nu]} \\
\delta_0 \Omega_{\mu}{}^{abc} &= D_{\mu} \eta^{abc} + \frac{4m_1}{3(d-3)} e_{\mu}{}^{[a} \eta^{bc]} \\
\delta_0 f_{\mu}{}^a &= D_{\mu} \xi^a + \eta_{\mu}{}^a + 4m_1 z_{\mu}{}^a \\
\delta_0 \omega_{\mu}{}^{ab} &= D_{\mu} \eta^{ab} - 2m_1 \eta_{\mu}{}^{ab}
\end{aligned} \tag{22}$$

corresponds to massless representation of AdS group (which differs from that of Poincare group [51]), while the other one just gives gauge invariant description of massive antisymmetric second rank tensor. In turn, in dS space one can set $m_1 = 0$. In this case the whole system also decomposes into two disconnected subsystems. One of them with the Lagrangian and gauge transformations:

$$\begin{aligned}
\mathcal{L}_0 &= -\frac{3}{4} \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} \Omega_{\mu}{}^{acd} \Omega_{\nu}{}^{bcd} + \frac{1}{4} \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \Omega_{\mu}{}^{abc} D_{\nu} \Phi_{\alpha\beta}{}^d + \\
&\quad -\frac{1}{6} C_{abc}{}^2 + \frac{1}{6} \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} C^{abc} D_{\mu} C_{\nu\alpha} + \\
&\quad + m_2 \left[\left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \Omega_{\mu}{}^{abc} C_{\nu\alpha} + \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} C^{abc} \Phi_{\mu\nu}{}^c \right]
\end{aligned} \tag{23}$$

$$\begin{aligned}
\delta_0 \Phi_{\mu\nu}{}^a &= D_{[\mu} z_{\nu]}{}^a + \eta_{\mu\nu}{}^a + \frac{4m_2}{(d-3)} e_{[\mu}{}^a \zeta_{\nu]}, & \delta_0 \Omega_{\mu}{}^{abc} &= D_{\mu} \eta^{abc} \\
\delta_0 C_{\mu\nu} &= D_{[\mu} \zeta_{\nu]} - 2m_2 z_{[\mu\nu]}, & \delta_1 C^{abc} &= 6m_2 \eta^{abc}
\end{aligned} \tag{24}$$

corresponds to massless representation of dS group, while the other one describes a so called partially massless spin 2 particle [45, 46, 47, 48, 49, 50, 30].

2.2 Gauge invariant objects

Having in our disposal explicit form of the gauge transformations we can construct gauge invariant curvatures for all eight fields (both physical and auxiliary):

$$\begin{aligned}
\mathcal{R}_{\mu\nu}{}^{abc} &= D_{[\mu} \Omega_{\nu]}{}^{abc} + \frac{4m_1}{3(d-3)} e_{[\mu}{}^{[a} \omega_{\nu]}{}^{bc]} + \frac{4m_2}{3(d-3)} e_{[\mu}{}^{[a} C_{\nu]}{}^{bc]} \\
\mathcal{T}_{\mu\nu\alpha}{}^a &= D_{[\mu} \Phi_{\nu\alpha]}{}^a - \Omega_{[\mu, \nu\alpha]}{}^a + \frac{2m_1}{3(d-3)} e_{[\mu}{}^a f_{\nu, \alpha]} + \frac{4m_2}{(d-3)} e_{[\mu}{}^a C_{\nu\alpha]} \\
\mathcal{F}_{\mu\nu}{}^{ab} &= D_{[\mu} \omega_{\nu]}{}^{ab} + 2m_1 \Omega_{[\mu, \nu]}{}^{ab} - \frac{2\tilde{m}_2}{(d-2)} e_{[\mu}{}^{[a} B_{\nu]}{}^{b]} \\
T_{\mu\nu}{}^a &= D_{[\mu} f_{\nu]}{}^a - \omega_{[\mu, \nu]}{}^a - 4m_1 \Phi_{\mu\nu}{}^a + \frac{4\tilde{m}_2}{(d-2)} e_{[\mu}{}^a B_{\nu]} \\
\mathcal{C}_{\mu}{}^{abc} &= D_{\mu} C^{abc} - 6m_2 \Omega_{\mu}{}^{abc} - \frac{2\tilde{m}_1}{(d-2)} e_{\mu}{}^{[a} B^{bc]} \\
\mathcal{C}_{\mu\nu\alpha} &= D_{[\mu} C_{\nu\alpha]} - C_{\mu\nu\alpha} + 2m_2 \Phi_{[\mu\nu, \alpha]} \\
\mathcal{B}_{\mu}{}^{ab} &= D_{\mu} B^{ab} + 4\tilde{m}_2 \omega_{\mu}{}^{ab} + \frac{4\tilde{m}_1}{3} C_{\mu}{}^{ab} \\
\mathcal{B}_{\mu\nu} &= D_{[\mu} B_{\nu]} - B_{\mu\nu} - 2\tilde{m}_2 f_{[\mu, \nu]} - 4\tilde{m}_1 C_{\mu\nu}
\end{aligned} \tag{25}$$

Similarly to massive spin 2 case, if we consider the most general Lagrangian quadratic in these curvatures and require that all higher derivatives terms were absent and kinetic terms were diagonal, we will still have some ambiguity in the choice of coefficients. In what follows we will use the following concrete form of free Lagrangian (see below):

$$\begin{aligned}\mathcal{L}_0 = & \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} [a_1 \mathcal{R}_{\mu\nu}{}^{abe} \mathcal{R}_{\alpha\beta}{}^{cde} + a_2 \mathcal{F}_{\mu\nu}{}^{ab} \mathcal{F}_{\alpha\beta}{}^{cd}] + \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} [a_3 \mathcal{C}_\mu{}^{acd} \mathcal{C}_\nu{}^{bcd} + a_4 \mathcal{B}_\mu{}^{ac} \mathcal{B}_\nu{}^{bc}] + \\ & + b_1 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \mathcal{R}_{\mu\nu}{}^{abc} T_{\alpha\beta}{}^d + b_2 \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} \mathcal{C}_\mu{}^{abc} \mathcal{B}_{\nu\alpha}\end{aligned}\quad (26)$$

$$\begin{aligned}a_1 &= -\frac{9}{512m_1^2}, & a_2 &= \frac{2}{3(d-3)}a_1 = -\frac{3}{256(d-3)m_1^2} \\ a_3 &= \frac{32(d-4)}{9(d-3)}a_1 = -\frac{(d-4)}{16(d-3)m_1^2}, & a_4 &= \frac{4(d-3)}{(d-2)}a_2 = -\frac{3}{64(d-2)m_1^2} \\ b_1 &= -\frac{1}{32m_1}, & b_2 &= \frac{1}{24\tilde{m}_1}\end{aligned}$$

2.3 Deformations

Our next task is to construct appropriate deformations for all gauge invariant objects. As in the case of massive spin 2 particle all we have to do with curvatures for mixed symmetry tensor is to replace $(A)dS$ covariant derivatives with the fully covariant ones. Using explicit form of free gauge transformations and free curvatures it is straightforward to find appropriate deformation for e/m field strength. Indeed, let us consider the following ansatz for such deformation:

$$\hat{F}_{\mu\nu} = a_0 \varepsilon^{ij} [\Omega_{[\mu}{}^{abc,i} \Omega_{\nu]}{}^{abc,j} + \alpha_1 \omega_{[\mu}{}^{ab,i} \omega_{\nu]}{}^{ab,j} + \alpha_2 C_{[\mu}{}^{ab,i} C_{\nu]}{}^{ab,j} + \alpha_3 B_{[\mu}{}^{a,i} B_{\nu]}{}^{a,j}]$$

Its variation under the η^{abc} transformations has the form:

$$\delta \hat{F}_{\mu\nu} = 2a_0 \varepsilon^{ij} [-D_{[\mu} \eta^{abc,j} \Omega_{\nu]}{}^{abc,i} - 2m_1 \alpha_1 \omega_{[\mu}{}^{ab,i} \eta_{\nu]}{}^{ab,j} + 6m_2 \alpha_2 C_{[\mu}{}^{ab,i} \eta_{\nu]}{}^{ab,j}]$$

Introducing appropriate correction:

$$\delta A_\mu = 2a_0 \varepsilon^{ij} \Omega_\mu{}^{abc,i} \eta^{abc,j}$$

we obtain:

$$\delta \hat{F}_{\mu\nu} = 2a_0 \varepsilon^{ij} [D_{[\mu} \Omega_{\nu]}{}^{abc,i} \eta^{abc,j} - 2m_1 \alpha_1 \omega_{[\mu}{}^{ab,i} \eta_{\nu]}{}^{ab,j} + 6m_2 \alpha_2 C_{[\mu}{}^{ab,i} \eta_{\nu]}{}^{ab,j}]$$

Comparing this expression with

$$2a_0 \varepsilon^{ij} \mathcal{R}_{\mu\nu}{}^{abc,i} \eta^{abc,j} = 2a_0 \varepsilon^{ij} [D_{[\mu} \Omega_{\nu]}{}^{abc,i} + \frac{4m_1}{(d-3)} e_{[\mu}{}^a \omega_{\nu]}{}^{bc,i} + \frac{4m_2}{(d-3)} e_{[\mu}{}^a C_{\nu]}{}^{bc,i}] \eta^{abc,j}$$

we see that we have to put

$$\alpha_1 = \frac{2}{(d-3)}, \quad \alpha_2 = -\frac{2}{3(d-3)}$$

Similarly, variation under the η^{ab} transformations:

$$\delta \hat{F}_{\mu\nu} = 2a_0 \varepsilon^{ij} [-\alpha_1 D_{[\mu} \eta^{ab,j} \omega_{\nu]}^{ab,i} + \frac{4m_1}{(d-3)} \Omega_{[\mu}^{abc,i} e_{\nu]}^a \eta^{bc,j} - 4\tilde{m}_2 \alpha_3 B_{[\mu}^{a,i} \eta_{\nu]}^{a,j}]$$

together with the following correction

$$\delta A_\mu = 2\alpha_1 a_0 \varepsilon^{ij} \omega_\mu^{ab,i} \eta^{ab,j}$$

give us

$$\delta \hat{F}_{\mu\nu} = 2a_0 \varepsilon^{ij} [\alpha_1 D_{[\mu} \omega_{\nu]}^{ab,i} \eta^{ab,j} + \frac{4m_1}{(d-3)} \Omega_{[\mu}^{abc,i} e_{\nu]}^a \eta^{bc,j} - 4\tilde{m}_2 \alpha_3 B_{[\mu}^{a,i} \eta_{\nu]}^{a,j}]$$

Comparing this expression with

$$2\alpha_1 a_0 \varepsilon^{ij} \mathcal{F}_{\mu\nu}^{ab,i} \eta^{ab,j} = 2\alpha_1 a_0 \varepsilon^{ij} [D_{[\mu} \omega_{\nu]}^{ab,i} + 2m_1 \Omega_{[\mu,\nu]}^{ab,i} - \frac{4\tilde{m}_2}{(d-2)} e_{[\mu}^a B_{\nu]}^{b,i}] \eta^{ab,j}$$

we obtain

$$\alpha_3 = -\frac{2}{(d-2)(d-3)}$$

Thus we obtain finally

$$\begin{aligned} \hat{F}_{\mu\nu} = & F_{\mu\nu} + a_0 \varepsilon^{ij} [\Omega_{[\mu}^{abc,i} \Omega_{\nu]}^{abc,j} + \frac{2}{(d-3)} \omega_{[\mu}^{ab,i} \omega_{\nu]}^{ab,j} - \\ & - \frac{2}{3(d-3)} C_{[\mu}^{ab,i} C_{\nu]}^{ab,j} - \frac{1}{2(d-3)(d-2)} B_{[\mu}^{a,i} B_{\nu]}^{a,j}] \end{aligned} \quad (27)$$

In this, if we supplement this deformation with the following corrections to e/m field transformations:

$$\delta A_\mu = 2a_0 \varepsilon^{ij} [\Omega_\mu^{abc,i} \eta^{abc,j} + \frac{2}{(d-3)} \omega_\mu^{ab,i} \eta^{ab,j}] \quad (28)$$

then under massive mixed tensor gauge transformations we obtain:

$$\delta \hat{F}_{\mu\nu} = 2a_0 \varepsilon^{ij} [\mathcal{R}_{\mu\nu}^{abc,i} \eta^{abc,j} + \frac{2}{(d-3)} \mathcal{F}_{\mu\nu}^{ab,i} \eta^{ab,j}] \quad (29)$$

Thus, following general procedure, we consider the following Lagrangian:

$$\begin{aligned} \mathcal{L} = & \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} [a_1 \hat{\mathcal{R}}_{\mu\nu}^{abe} \hat{\mathcal{R}}_{\alpha\beta}^{cde} + a_2 \hat{\mathcal{F}}_{\mu\nu}^{ab} \hat{\mathcal{F}}_{\alpha\beta}^{cd}] + \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} [a_3 \hat{\mathcal{C}}_\mu^{acd} \hat{\mathcal{C}}_\nu^{bcd} + a_4 \hat{\mathcal{B}}_\mu^{ac} \hat{\mathcal{B}}_\nu^{bc}] + \\ & + b_1 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \hat{\mathcal{R}}_{\mu\nu}^{abc} \hat{T}_{\alpha\beta}^d + \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} b_2 \hat{\mathcal{C}}_\mu^{abc} \hat{\mathcal{B}}_{\nu\alpha} - \frac{1}{4} \hat{F}_{\mu\nu}^2 \end{aligned} \quad (30)$$

2.4 Gauge invariance

Now we have to calculate variations of this Lagrangian in the first non-trivial approximation and to adjust parameters so that all such variations vanish on-shell. This in turn would imply that they can be compensated by appropriate corrections to gauge transformations.

η^{abc} transformations. In this case we obtain the following variations of our Lagrangian:

$$\begin{aligned}\delta\mathcal{L} = & -a_0\varepsilon^{ij}F^{\mu\nu}\mathcal{R}_{\mu\nu}{}^{cde,i}\eta^{cde,j} + 8a_1e_0\varepsilon^{ij}\mathcal{R}_{\mu\nu}{}^{cde,i}F^{cd}\eta^{\mu\nu e,j} + \\ & + 4a_1e_0\varepsilon^{ij}\left\{\begin{smallmatrix}\mu\nu\\ab\end{smallmatrix}\right\}[-4\mathcal{R}_{\mu\nu}{}^{ace,i}\eta^{bde,j} + \mathcal{R}_{\mu\nu}{}^{abe,i}\eta^{cde,j}]F^{cd} + \\ & + 6b_1e_0\varepsilon^{ij}\left\{\begin{smallmatrix}\mu\nu\\ab\end{smallmatrix}\right\}[T_{\mu\nu}{}^{a,i}\eta^{bcd,j} + T_{\mu\nu}{}^{c,i}\eta^{abd,j}]F^{cd}\end{aligned}\quad (31)$$

All terms in the second and the third lines clearly vanish on-shell being proportional to $\Phi_{\mu\nu}{}^a$ and $\omega_\mu{}^{ab}$ equations correspondingly. Now using explicit expressions for free curvatures it is not hard to check that the following identity holds:

$$\mathcal{R}_{[\mu\nu,\alpha\beta]}{}^a = -D_{[\mu}\mathcal{T}_{\nu\alpha\beta]}{}^a - \frac{2m_1}{3(d-3)}e_{[\mu}{}^aT_{\nu\alpha,\beta]} - \frac{4m_2}{(d-3)}e_{[\mu}{}^a\mathcal{C}_{\nu\alpha\beta]}\quad (32)$$

Thus $\mathcal{R}_{[\mu\nu,\alpha\beta]}{}^a$ vanish on-shell, in this the following consequence appears:

$$\mathcal{R}_{[\mu\nu,\alpha\beta]}{}^a = 0 \quad \implies \quad \eta^{abc}F^{de}\mathcal{R}_{ab,cde} = \frac{1}{3}F^{ab}\eta^{cde}\mathcal{R}_{ab,cde}$$

Thus if we put $a_0 = \frac{8e_0a_1}{3}$ then all variations vanish on-shell and can be compensated with the corresponding corrections to gauge transformations. Again we restrict ourselves with the corrections for physical fields only which have the following form:

$$\delta_1\Phi_{\mu\nu}{}^{a,i} = 2a_0\varepsilon^{ij}[2F_{[\mu}{}^b\eta_{\nu]}{}^{ab,j} - \frac{1}{(d-3)}e_{[\mu}{}^a(F\eta)_{\nu]}{}^j]\quad (33)$$

η^{ab} transformations. Here we obtain the following variations:

$$\begin{aligned}\delta_\eta\mathcal{L} = & -2a_2e_0\varepsilon^{ij}\left\{\begin{smallmatrix}\mu\nu\\ab\end{smallmatrix}\right\}[4\mathcal{F}_{\mu\nu}{}^{ac,i}\eta^{bd,j} - \mathcal{F}_{\mu\nu}{}^{ab,i}\eta^{cd,j}]F^{cd} - \\ & - \frac{2a_0}{(d-3)}\varepsilon^{ij}F^{\mu\nu}\mathcal{F}_{\mu\nu}{}^{ab,i}\eta^{ab,j} + 8a_2e_0\varepsilon^{ij}\mathcal{F}_{\mu\nu}{}^{ab,i}F^{ab}\eta^{\mu\nu,j}\end{aligned}$$

Terms in the first line vanish on-shell being proportional to free $f_\mu{}^a$ equations. As for the second line, it is not hard to check that the following identity holds:

$$\mathcal{F}_{[\mu\nu,\alpha]}{}^a = -D_{[\mu}T_{\nu\alpha]}{}^a - 4m_1\mathcal{T}_{\mu\nu\alpha}{}^a - \frac{4\tilde{m}_2}{(d-2)}e_{[\mu}{}^a\mathcal{B}_{\nu\alpha]}\quad (34)$$

This means that $\mathcal{F}_{[\mu\nu,\alpha]}{}^a$ vanish on-shell. Thus if we put $a_0 = 4a_2e_0(d-3)$ then all variations vanish on-shell. Comparing this relation with $a_0 = \frac{8e_0a_1}{3}$ we see that $a_2 = \frac{2a_1}{3(d-3)}$ and this explains our choice of concrete form for the free Lagrangian. Corrections to gauge transformations for physical fields look as follows:

$$\delta f_\mu{}^{a,i} = \frac{2a_0}{(d-3)}\varepsilon^{ij}[2F_\mu{}^b\eta^{ab,j} - \frac{1}{(d-2)}e_\mu{}^a(F\eta)^j]\quad (35)$$

ξ^a and Λ transformations. In these cases we obtain simply:

$$\delta\mathcal{L} = b_1e_0\varepsilon^{ij}\left\{\begin{smallmatrix}\mu\nu\alpha\beta\\abcd\end{smallmatrix}\right\}\mathcal{R}_{\mu\nu}{}^{abc,i}F_{\alpha\beta}\xi^{d,j} + b_2e_0\varepsilon^{ij}\left\{\begin{smallmatrix}\mu\nu\alpha\\abc\end{smallmatrix}\right\}\mathcal{C}_\mu{}^{abc,i}F_{\nu\alpha}\Lambda^j\quad (36)$$

and these variations can be compensated by:

$$\delta_2\Phi_{\mu\nu}{}^{a,i} = -8b_1e_0\varepsilon^{ij}F_{\mu\nu}\xi^{a,j}, \quad \delta C_{\mu\nu}{}^i = 6b_2e_0\varepsilon^{ij}F_{\mu\nu}\Lambda^j\quad (37)$$

2.5 Results

Thus we have seen that the following Lagrangian:

$$\begin{aligned} \mathcal{L} = & \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} [a_1 \hat{\mathcal{R}}_{\mu\nu}{}^{abe} \hat{\mathcal{R}}_{\alpha\beta}{}^{cde} + a_2 \hat{\mathcal{F}}_{\mu\nu}{}^{ab} \hat{\mathcal{F}}_{\alpha\beta}{}^{cd}] + \left\{ \begin{smallmatrix} \mu\nu \\ ab \end{smallmatrix} \right\} [a_3 \hat{\mathcal{C}}_{\mu}{}^{acd} \hat{\mathcal{C}}_{\nu}{}^{bcd} + a_4 \hat{\mathcal{B}}_{\mu}{}^{ac} \hat{\mathcal{B}}_{\nu}{}^{bc}] + \\ & + b_1 \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} \hat{\mathcal{R}}_{\mu\nu}{}^{abc} \hat{T}_{\alpha\beta}{}^d + \left\{ \begin{smallmatrix} \mu\nu\alpha \\ abc \end{smallmatrix} \right\} b_2 \hat{\mathcal{C}}_{\mu}{}^{abc} \hat{\mathcal{B}}_{\nu\alpha} - \frac{1}{4} \hat{F}_{\mu\nu}{}^2 \end{aligned} \quad (38)$$

is indeed gauge invariant in the linear approximation provided we supplement it with the following corrections to gauge transformations:

$$\begin{aligned} \delta A_{\mu} &= 2a_0 \varepsilon^{ij} [\Omega_{\mu}{}^{abc,i} \eta^{abc,j} + \frac{2}{(d-3)} \omega_{\mu}{}^{ab,i} \eta^{ab,j}] \\ \delta \Phi_{\mu\nu}{}^{a,i} &= 2a_0 \varepsilon^{ij} [2F_{[\mu}{}^b \eta_{\nu]}{}^{ab,j} - \frac{1}{(d-3)} e_{[\mu}{}^a (F\eta)_{\nu]}{}^j] - 8b_1 e_0 \varepsilon^{ij} F_{\mu\nu} \xi^{a,j} \\ \delta f_{\mu}{}^{a,i} &= \frac{2a_0}{(d-3)} \varepsilon^{ij} [2F_{\mu}{}^b \eta^{ab,j} - \frac{1}{(d-2)} e_{\mu}{}^a (F\eta)^j] \\ \delta C_{\mu\nu}{}^i &= 6b_2 e_0 \varepsilon^{ij} F_{\mu\nu} \Lambda^j \end{aligned} \quad (39)$$

As we have seen from the formulas above, electric charge $e_0 \sim a_0 m_1$ so it is impossible to take (partially) massless limit $m_1 \rightarrow 0$ in de Sitter space without switching off minimal e/m interactions. In this, nothing prevent us from considering (partially) massless limit $m_2 \rightarrow 0$ in anti de Sitter space. In this limit we obtain the following simple Lagrangian:

$$\mathcal{L} = \left\{ \begin{smallmatrix} \mu\nu\alpha\beta \\ abcd \end{smallmatrix} \right\} [a_1 \hat{\mathcal{R}}_{\mu\nu}{}^{abe} \hat{\mathcal{R}}_{\alpha\beta}{}^{cde} + a_2 \hat{\mathcal{F}}_{\mu\nu}{}^{ab} \hat{\mathcal{F}}_{\alpha\beta}{}^{cd} + b_1 \hat{\mathcal{R}}_{\mu\nu}{}^{abc} \hat{T}_{\alpha\beta}{}^d] - \frac{1}{4} \hat{F}_{\mu\nu}{}^2 \quad (40)$$

where:

$$\begin{aligned} \hat{F}_{\mu\nu} &= F_{\mu\nu} + a_0 \varepsilon^{ij} [\Omega_{[\mu}{}^{abc,i} \Omega_{\nu]}{}^{abc,j} + \frac{2}{(d-3)} \omega_{[\mu}{}^{ab,i} \omega_{\nu]}{}^{ab,j}] \\ \mathcal{R}_{\mu\nu}{}^{abc} &= D_{[\mu} \Omega_{\nu]}{}^{abc} + \frac{4m_1}{3(d-3)} e_{[\mu}{}^a \omega_{\nu]}{}^{bc} \\ \mathcal{F}_{\mu\nu}{}^{ab} &= D_{[\mu} \omega_{\nu]}{}^{ab} + 2m_1 \Omega_{[\mu,\nu]}{}^{ab} \\ T_{\mu\nu}{}^a &= D_{[\mu} f_{\nu]}{}^a - \omega_{[\mu,\nu]}{}^a - 4m_1 \Phi_{\mu\nu}{}^a \end{aligned} \quad (41)$$

Conclusion

Thus we have seen that using frame-like gauge invariant formulation it is indeed possible to extend Fradkin-Vasiliev procedure to the case of electromagnetic interactions of massive particles. We constructed two explicit examples: spin 2 and simplest mixed symmetry tensor. These two cases are just the simplest representatives of two general class of fields, namely completely symmetric and mixed symmetry ones, and it is clear that the results obtained admit straightforward generalization to higher spins as well.

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